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Covariant Quantization of D-branes

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ABSTRACT

We have found that κ-symmetry allows a covariant quantization provided the ground state of the theory is strictly massive. For D-p-branes a Hamiltonian analysis is performed to explain the existence of a manifestly supersymmetric and Lorentz covariant description of the BPS states of the theory. The covariant quantization of the D-0-brane is presented as an example.

1 Introduction

Extended objects with global supersymmetry have local κ-symmetry. This symmetry is difficult to quantize in Lorentz covariant gauges keeping finite number of fields in the theory. A revival of interest to κ-symmetric objects is due to the recent discovery of D-p-branes [1] and D-p-brane actions [2, 3, 4, 5].

Before the choice of the gauge is made the problem with covariant quantization of κ-symmetry can be seen as the impossibility to disentangle covariantly the combination of the first and second class fermionic constraints. In the past the quantization of the superparticle, of the supersymmetric string, and of the supermembrane was performed only in the light-cone gauge for spinors

$$\Gamma^{+}\theta = 0 \ . \tag{1}$$

In this gauge κ-symmetry is fixed leaving only the second class fermionic constraints whose Poisson brackets are invertible even for the massless ground state. In covariant gauges the major problem is that the constraints are not invertible.

Recently a covariant gauge fixing κ-symmetry of D-branes has been discovered [3]. The fermionic gauge is of the form

$$\theta_2 = 0 \ , \tag{2}$$

where θ_2 is one of the chiral spinors of the 10d theory. Moreover, since there is a duality between the D-1 brane and the fundamental IIB string, a gauge-fixing of the fundamental string in covariant gauge has been achieved in [3] by passing. Does it mean that the previous attempt to covariantly quantize the Green-Schwarz string missed the point, or something else happened? Is the existence of covariant gauges a special property of D-p-branes only, or they exist for all p-branes? To address these issues one has to take into account that the covariant quantization performed in [3] also used the so-called static gauges $X^m = \sigma^m$ for fixing the bosonic reparametrization symmetry. The total picture of covariant quantization of κ-symmetry with the use of static gauges is difficult to analyze.

Here we will first switch to a Hamiltonian form of the theory which will allow us to study the issues in quantization of κ-symmetry before a choice of the reparametrization fixing gauge is made. We will analyze the quantization of κ-symmetric theories and we will establish connection of the quantized theory with $d = 10$, $N = 2$ supersymmetry algebra with central extensions:

$$\{Q_\alpha,\ Q_\beta\} \quad = \quad 2(C\Gamma^m)_{\alpha\beta}\Big(\mathbf{P}_m + \frac{Q^N_m}{2\pi\alpha'}\Big) \ , \tag{3}$$

$$\{\tilde{Q}_\alpha,\ \tilde{Q}_\beta\} \quad = \quad 2(C\Gamma^m)_{\alpha\beta}\Big(\mathbf{P}_m - \frac{Q^N_m}{2\pi\alpha'}\Big) \ , \tag{4}$$

$$\{Q_\alpha,\ \tilde{Q}_\beta\} \ = 2 \ \sum_A (C\Gamma^A)_{\alpha\beta}\, T_{(p)} \frac{Q^R_A}{p!} \ . \tag{5}$$

We present here the supersymmetry algebra in the form given in [1]. Here Q^N and Q^R are NS-NS and R-R charges and A runs over antisymmetrized products of gamma matrices. We will find the covariant mass formula for the quantized D-p-branes.

The quantization of D-p-branes in static gauges [3] leads to complicated non-linear actions. In this paper we will perform a covariant quantization of the D-0-brane which will give a simple quadratic action.

2 Main results

We have found that the covariant quantization of D-branes is consistent and that the ground state $|\Psi\rangle$ of the system has a non-vanishing mass

$$M^2|\Psi\rangle = -(\Gamma^m \mathbf{P}_m)^2|\Psi\rangle = T^2 \left(\det(G+\mathcal{F})_{ab} + P^a G_{ab} P^b\right)|\Psi\rangle \ , \tag{6}$$

$$M^2 > 0 \ . \tag{7}$$

Here $\mathbf{P}_m$ is the momentum conjugate to the coordinate X^m and the eigenvalue of the operator $-(\Gamma^m \mathbf{P}_m)^2$ is given by the positive definite expression $(T^2 \det(G+\mathcal{F})_{ab} + P^a G_{ab} P^b)$. T is the D-brane tension, the index a runs over the space components of the brane, $G_{ab}, \mathcal{F}_{ab}$ are the space part of metric induced on the brane and the 2-form, respectively, and P^a is the momentum conjugate to the vector field on the brane. Both the metric G_{ab} and the 2-form $\mathcal{F}_{ab}$ are manifestly supersymmetric under the N=1 part of the full N=2 global supersymmetry.

For example, for the covariantly quantized D-2-brane we find from (6) that the mass of the ground state is

$$M^2_{(D2)} = T^2\{\Pi^m, \Pi^n\}^2 + (\mathcal{F}_{12})^2 + P^r G_{rs} P^s, \qquad m,n = 0,1\ldots,9; \quad r,s = 1,2. \tag{8}$$

Here the square of the Poisson bracket is defined as,

$$\{\Pi^m, \Pi^n\}^2 \equiv [\left(X^m{}_{,r} - \bar\lambda \Gamma^m \lambda_{,r}\right)\left(X^n{}_{,s} - \bar\lambda \Gamma^n \lambda_{,s}\right) - (m \to n)]^2 \ , \tag{9}$$

and λ is one of the chiral d=10 spinors which remains in the theory after κ-symmetry is gauge-fixed covariantly. This is a Lorentz covariant generalization of the mass operator

$$M^2_{(M2)} = \{X^i, X^j\}^2 - \bar\theta \Gamma_- \Gamma_i \{\theta, X^i\}, \qquad i,j = 1,\ldots 9, \tag{10}$$

which appears in the quantization of the M-2 supermembrane [6] in the light-cone gauge [7, 8]. In the form in which r,s are matrix indices this expression has been used in M(atrix)model [9].

The static gauge for the D-2 brane $X^1 = \sigma, X^2 = \rho$ corresponds to

$$\{\Pi^1, \Pi^2\} = (X^1)_{,\sigma}(X^2)_{,\rho} + \ldots = 1 + \ldots \tag{11}$$

This makes the constant part of the square of the momentum on the ground state non-vanishing as long as the tension T is non-vanishing. Thus the covariant quantization of the D-2-brane performed in [3] indeed confirms our general conclusion that the ground state of a D-brane has a non-vanishing mass as long as the tension of the D-brane is non-vanishing.

This observation also solves the apparent paradox with the covariant quantization of κ-symmetry of the IIB fundamental string in [3]. For the D-1-brane we get

$$M^2_{(D1)} = T^2(\Pi^1)^2 + P^1 G_{11} P^1 \ . \tag{12}$$

The ground state of the D-1-brane is massive. The technical reason for this (in Lorentz covariant gauge for spinors and the static gauge $X^1 = \sigma$ for reparametrization symmetry [3]) is that $(\Pi^1)^2 = [(X^1)_{,\sigma}]^2 + \ldots = 1 + \cdots$. This corresponds to the D-string wrapped around the circle. In case of a fundamental string, the covariant gauge-fixing proposed in [3] also uses a static gauge. Therefore *the massless state of the fundamental GS string is projected out*, and in this way the two quantized string theories, D-1-brane and type IIB fundamental strings are dual to each other. For the D-0-brane the mass formula (6) cannot be applied since there are no space directions. However, the mass formula in this case is simply $M^2 = Z^2$, where Z equals the tension of the D-0-brane.

We will proceed with the derivation of the results stated above.

3 Irreducible κ-symmetry on D-branes

The κ-symmetric D-brane action in the flat background geometry[1] consists of the Born-Infeld-Nambu-Goto term S_1 and Wess-Zumino term S_2:

$$S_{\rm DBI} + S_{\rm WZ} = T\left(-\int d^{p+1}\sigma \sqrt{-\det(G_{\mu\nu} + \mathcal{F}_{\mu\nu})} + \int \Omega_{p+1}\right) \ . \tag{13}$$

Here T is the tension of the D-brane, $G_{\mu\nu}$ is the manifestly supersymmetric induced world-volume metric

$$G_{\mu\nu} = \eta_{mn}\Pi^m_\mu \Pi^n_\nu \ , \qquad \Pi^m_\mu = \partial_\mu X^m - \bar\theta\Gamma^m\partial_\mu\theta \ , \tag{14}$$

and $\mathcal{F}_{\mu\nu}$ is a manifestly supersymmetric Born-Infeld field strength (for p even) [2]

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} - b_{\mu\nu} = \Big[\partial_\mu A_\nu - \bar\theta\Gamma_{11}\Gamma_m\partial_\mu\theta\Big(\partial_\nu X^m - \frac{1}{2}\bar\theta\Gamma^m\partial_\nu\theta\Big)\Big] - (\mu\leftrightarrow\nu) \ . \tag{15}$$

When p is odd, Γ_{11} is replaced by $\tau_3 \otimes I$. The action has global supersymmetry

$$\delta_\epsilon\theta = \epsilon, \qquad \delta_\epsilon X^m = \bar\epsilon\Gamma^m\theta \ . \tag{16}$$

[1] We use notation of [3].

[2] We define spinors for even p as $\theta = \theta_1 + \theta_2$ where $\theta_1 \equiv \frac{1}{2}(1+\Gamma_{11})\theta$ and $\theta_2 \equiv \frac{1}{2}(1-\Gamma_{11})\theta$.

and local κ-supersymmetry:

$$\delta X^m = \bar{\theta}\Gamma^m \delta\theta = -\delta\bar{\theta}\Gamma^m\theta, \qquad \delta\bar{\theta} = \bar{\kappa}(1+\Gamma), \tag{17}$$

and

$$\Gamma = e^{\frac{a}{2}}\Gamma'_{(0)} e^{-\frac{a}{2}}\,, \tag{18}$$

where

$$a = \begin{cases} +\frac{1}{2} Y_{jk}\gamma^{jk}\Gamma_{11} & \text{IIA} \ , \\ -\frac{1}{2} Y_{jk}\gamma^{jk}\sigma_3 \otimes 1 & \text{IIB} \ . \end{cases} \tag{19}$$

Here $\Gamma'_{(0)}$ is the product structure, independent on the BI field, $(\Gamma'_{(0)})^2 = 1, tr\,\Gamma'_{(0)} = 0)$. All dependence on BI field $\mathcal{F} =$ "tan"Y is in the exponent [10]. The matrix Γ_{11} in IIA and $\sigma_3 \otimes\ 1$ in IIB theory anticommute with $\Gamma'_{(0)}$ and with Γ. Therefore in the basis where Γ_{11} and $\sigma_3 \otimes\ 1$ are diagonal, $\Gamma'_{(0)}$ and Γ are off-diagonal.

$$\Gamma'_{(0)} = \begin{pmatrix} 0 & \hat{\gamma} \\ \hat{\gamma}^{-1} & 0 \end{pmatrix} . \tag{20}$$

We introduced a 16×16-dimensional matrix $\hat{\gamma}$ which does not depend on BI field. Now the κ-symmetry generator can be presented in a useful off-diagonal form

$$\Gamma = \begin{pmatrix} 0 & \hat{\gamma}e^{\hat{a}} \\ (\hat{\gamma}e^{\hat{a}})^{-1} & 0 \end{pmatrix} . \tag{21}$$

where

$$\hat{a} = \begin{cases} +\frac{1}{2} Y_{jk}\gamma^{jk} & \text{IIA} \ , \\ -\frac{1}{2} Y_{jk}\gamma^{jk} & \text{IIB} \ . \end{cases} \tag{22}$$

The fact that Γ is off-diagonal and that the matrix $\gamma e^{\hat{a}}$ is invertible is quite important and the significance of this was already discussed in [3, 10]. In particular this allows us to consider only irreducible κ-symmetry transformations by imposing a Lorentz covariant condition on $\bar{\kappa}$ of the form

$$\bar{\kappa}_1 = 0 \qquad \bar{\kappa}_2 \neq 0 \qquad \text{IIA} \tag{23}$$

$$\bar{\kappa}_2 = 0 \qquad \bar{\kappa}_1 \neq 0 \qquad \text{IIB} \tag{24}$$

In this way we have an irreducible 16-dimensional κ-symmetry since the matrix $\hat{\gamma}e^{\hat{a}}$ is invertible, acting as

$$\delta\bar{\theta}_1 = \bar{\kappa}_2\hat{\gamma}e^{\hat{a}} \qquad \delta\bar{\theta}_2 = \bar{\kappa}_2 \qquad \delta X^m = -\bar{\kappa}_2\Gamma^m\theta_2 \qquad \text{IIA} \tag{25}$$

$$\delta\bar{\theta}_1 = \bar{\kappa}_1 \qquad \delta\bar{\theta}_2 = \bar{\kappa}_1(\hat{\gamma}e^{\hat{a}})^{-1} \qquad \delta X^m = -\bar{\kappa}_1\Gamma^m\theta_1 \qquad \text{IIB} \tag{26}$$

4 Fermionic constraints prior to gauge-fixing

The Hamiltonian analysis of supersymmetric extended objects with κ-symmetry requires the knowledge of the fermionic constraints. We split the worldvolume coordinates σ^μ into time $\sigma^0 = \tau$ and space part σ^a where $a = 1, \ldots, p$. In the first approximation we will neglect the space-dependence on σ^a of spinors θ.

To find fermionic constraints we observe that our κ-symmetric D-p-brane actions depend on the following combinations of the time derivatives of the fields:

$$L_{\mathrm{DBI}}\left(\dot{X}^m - \bar{\theta}_1\Gamma^m\dot{\theta}_1 - \bar{\theta}_2\Gamma^m\dot{\theta}_2,\ \dot{A}_a - [\bar{\theta}_1\Gamma^m\dot{\theta}_1 - \bar{\theta}_2\Gamma^m\dot{\theta}_2]\Pi_{ma}\right) - L_{\mathrm{WZ}}(\bar{\theta}_1 T_{\mathrm{WZ}}\dot{\theta}_2 + c.c.)\ , \tag{27}$$

and $T_{\mathrm{WZ}} \equiv \Gamma^A Z_A$ in the WZ term and Γ^A are given by an odd number of antisymmetrized Γ-matrices in IIA theory and an even number in IIB case.

We introduce canonical momenta $\mathbf{P}_m, P^0, P^a, P_{\theta_1}, P_{\theta_2}$ to $X^m, A_0, A_a, \theta_1, \theta_2$. The fermionic constraints follow

$$\bar{\Phi}_1 = \bar{P}_{\theta_1} + \bar{\theta}_1(\mathbf{P}_m + P^a\Pi_a^m)\Gamma^m + \bar{\theta}_2\Gamma^M Z_M\ , \tag{28}$$

$$\bar{\Phi}_2 = \bar{P}_{\theta_2} + \bar{\theta}_2(\mathbf{P}_m - P^a\Pi_a^m)\Gamma^m + \bar{\theta}_1\Gamma^M Z_M\ . \tag{29}$$

These 32 constraints can be shown to represent 16 first class constraints and 16 second class ones. The Poisson brackets of these constraints are given by (terms with possible derivatives of the delta functions $\delta^p(\sigma^a - \tilde{\sigma}^a)$ are omitted)

$$\{\Phi_1(\tau,\sigma^a), \Phi_1(\tau,\tilde{\sigma}^a)\} = 2(\mathbf{P}_m + P^a\Pi_a^m)C\Gamma^m\delta^p(\sigma^a - \tilde{\sigma}^a) + \ldots \tag{30}$$

$$\{\Phi_2(\tau,\sigma^a), \Phi_2(\tau,\tilde{\sigma}^a)\} = 2(\mathbf{P}_m - P^a\Pi_a^m)C\Gamma^m\delta^p(\sigma^a - \tilde{\sigma}^a) + \ldots \tag{31}$$

$$\{\Phi_1(\tau,\sigma^a), \Phi_2(\tau,\tilde{\sigma}^a)\} = 2\Gamma^A Z_A\delta^p(\sigma^a - \tilde{\sigma}^a) + \ldots \tag{32}$$

These brackets realize d=10, N=2 supersymmetry algebra with central extensions. The R-R charges in this algebra Z_A are due to the structure of the WZ part of the action. For example, in $p = 0$ case we have $\Gamma^A Z_A = Z$, where Z is the mass of the D-0-particle. In $p = 1$ we get $\Gamma^A Z_A = \Gamma^m\Pi_\sigma^m$, for $p = 2$ this term is $\Gamma^A Z_A = \Gamma^{m_1}\Gamma^{m_2}\epsilon^{ab}[\Pi_{am_1}\Pi_{bm_2} + \mathcal{F}_{ab}]$, $a = 1, 2$, etc. The terms with the $P^a\Pi_a^m$ could be considered as defining the NS-NS charge.

The problem of covariant quantization of the fundamental string was the impossibility to disentangle these constraints into the first class and the second class covariantly. For the D-branes the situation is different due to the presence of the R-R charges which appear in the bracket of Φ_1 and Φ_2 in (32). We will see later that the reparametrization constraints in presence of R-R central charges are such that the bracket for e. g. $\{\Phi_1,\ \Phi_1\}$ is invertible.

5 Gauge-fixed κ-symmetry

From now on we will study the possibilities to quantize these actions. One of the possibilities which we will pursue here is to immediately gauge-fix κ-symmetry by choosing the gauge for theta's. Here again we will follow [3] and simply take

$$\theta_2 = 0 \qquad \text{IIA} \qquad \theta_1 \equiv \lambda \tag{33}$$

$$\theta_1 = 0 \qquad \text{IIB} \qquad \theta_2 \equiv \lambda \tag{34}$$

Note that our choice of irreducible κ-symmetry (which is not unique) was made here with the purpose to explicitly eliminate θ_2 (θ_1) in IIA (IIB) case using $\delta\bar\theta_2 = \bar\kappa_2$ ($\delta\bar\theta_1 = \bar\kappa_1$). The gauge-fixed action has one particularly useful property: the Wess-Zumino term vanishes in this gauge [3]. We are left with the reparametrization invariant action:

$$S_{\kappa-\text{fixed}} = -\int d^{p+1}\sigma\sqrt{-\det(G_{\mu\nu} + \mathcal{F}_{\mu\nu})}\ , \tag{35}$$

$$G_{\mu\nu} = \eta_{mn}\Pi^m_\mu \Pi^n_\nu\ , \qquad \Pi^m_\mu = \partial_\mu X^m - \bar\lambda\Gamma^m\partial_\mu\lambda\ , \tag{36}$$

$$\mathcal{F}_{\mu\nu} = [\partial_\mu A_\nu - \bar\lambda\Gamma_m\partial_\mu\lambda\left(\partial_\nu X^m - \frac{1}{2}\bar\lambda\Gamma^m\partial_\nu\lambda\right)] - (\mu \leftrightarrow \nu)\ . \tag{37}$$

The justification of this procedure can be done either by showing that the remaining action does not have fermionic gauge symmetries anymore (which was done in [10]) or by the study of the Hamiltonian of the gauge-fixed theory and the constraints. In [3] the remaining reparametrization symmetry of the theory was gauge-fixed by choosing a static gauge. The resulting gauge-fixed action does not have fermionic degeneracy and this also shows that the choice of a gauge-fixing made in [3] is acceptable.

We will take from now on a different approach and study the Hamiltonian of the theory after κ-symmetry is gauge-fixed.

6 Hamiltonian structure of the theory

The set of canonical momenta and coordinates of the theory in (35) includes $(\mathbf{P}_m, X^m)$, (P^0, A_0) , (P^a, A_a) , (P_λ, λ). All expressions are relatively simple if as before we neglect in the first approximation terms with worldvolume space derivatives on spinors $\frac{\partial\lambda}{\partial\sigma^a}$. The phase space action (35) can be brought to the canonical form

$$L = p_i \partial_0 q^i + \xi^\alpha t_\alpha(p,q)\ , \tag{38}$$

where $t_\alpha(p,q)$ are some constraints of the first and second kind.

$$\begin{aligned} L_{\text{can}} \quad &= \mathbf{P}_m\partial_0 X^m + P^a F_{0a} + \bar P_\lambda \partial_0\lambda \\ &-\xi(\mathbf{P}_m\mathbf{P}^m + P^a G_{ab} P^b + T^2\det[(G+\mathcal{F})_{ab}]) - \xi_{\text{BI}} P^0 \\ &-\xi^a\left(\mathbf{P}_m \Pi^m_a + P^b\mathcal{F}_{ab}\right) + \left(\bar P_\lambda + \bar\lambda(\mathbf{P}_m + P^a\Pi^m_a)\Gamma^m\right)\psi + \ldots . \end{aligned} \tag{39}$$

Recently a Hamiltonian analysis of the bosonic part of the D-brane action was performed in [11]. Our κ-symmetry-gauge-fixed action is a supersymmetric generalization of the theory, studied in [11]. Indeed we have one manifest 16-dimensional global supersymmetry

$$\delta_\epsilon \lambda = \epsilon, \qquad \delta_\epsilon X^m = \bar\epsilon \Gamma^m \lambda \ . \tag{40}$$

The second 16-dimensional global supersymmetry exists since the gauge $\theta_2 = 0$ required $\kappa_2 + \epsilon_2 = 0$ and $\delta\bar\theta_1 = -\bar\epsilon_2 \hat\gamma e^{\hat a}$ in IIA theory and analogous in IIB case. At this stage it is important that we have a manifestly realized 16-dimensional supersymmetry. This allows us to use the structure of the bosonic Hamiltonian theory [11] and perform an N=1 supersymmetrization of it. The dots contain terms with space derivatives on spinors $\frac{\partial\lambda}{\partial\sigma^a}$. There is also a secondary "Gauss law" constraint [11]. The bosonic constraints related to the reparametrization gauge symmetry, the time reparametrization and space reparametrization constraints (with Lagrange multipliers ξ, ξ_a) are

$$t = \mathbf{P}_m \mathbf{P}^m + P^a G_{ab} P^b + T^2 \det[(G+\mathcal{F})_{ab}] \ , \tag{41}$$

$$t_a = \mathbf{P}_m \Pi^m_a + P^b \mathcal{F}_{ab} \ . \tag{42}$$

The constraint related to the abelian gauge symmetry (with the Lagrange multiplier $\xi_{\rm BI}$) is

$$t_{\rm BI} = P^0 \ . \tag{43}$$

These constraints are not much different from the constraints in the bosonic theory [11]. The new feature here is the presence of 16 fermionic constraints:

$$\bar\Phi_\lambda = \bar P_\lambda + \bar\lambda(\mathbf{P}_m + P^a \Pi^m_a)\Gamma^m \ . \tag{44}$$

If the gauge-fixing of the previous section is correct, we should find out that the fermionic constraints are second class, and there is no fermionic gauge symmetry left. For this to happen, the Poisson bracket of fermionic constraints has to be invertible when other constraints are imposed. Thus we have to calculate

$$\{\Phi_\lambda(\tau,\sigma^a), \Phi_\lambda(\tau,\tilde\sigma^a)\} = (\mathbf{P}_m + P^a \Pi^m_a) C \Gamma^m \delta^p(\sigma^a - \tilde\sigma^a) + \dots \tag{45}$$

Here ... stands for terms with derivatives of the $\delta^p(\sigma^a - \tilde\sigma^a)$-function and terms with space derivatives on spinors. The square of the matrix in the right-hand side of the constraint is

$$[(\mathbf{P}_m + P^a \Pi^m_a)\Gamma^m]^2 = \mathbf{P}_m \mathbf{P}^m + P^a G_{ab} P^b + 2\mathbf{P}_m P^a \Pi^m_a \ . \tag{46}$$

When the reparametrization constraints are imposed, we get

$$([(\mathbf{P}_m + P^a \Pi^m_a) C \Gamma^m]^2)_{T=T_a=0} = \mathbf{P}_m \mathbf{P}^m + P^a G_{ab} P^b = -T^2 \det[(G+\mathcal{F})_{ab}] \ . \tag{47}$$

This is quite remarkable: *the invertibility of the second class constraints* for D-branes quantized in the *Lorentz covariant gauge* relies on 2 basic facts: The tension has to be non-vanishing and the space part of the determinant of the BI action has to be non-vanishing,

$$T \neq 0 \ , \tag{48}$$

$$\det[(G+\mathcal{F})_{ab}] \neq 0 \ . \tag{49}$$

To understand the meaning of this restriction on the theory, we may consider the physical states satisfying all first class constraints

$$t|\Psi\rangle = t_a|\Psi\rangle = t_{\rm BI}|\Psi\rangle = 0 \ . \tag{50}$$

The mass of any such state we define by the eigenvalue of the square of the momentum

$$M^2|\Psi\rangle = -\mathbf{P}_m\mathbf{P}^m|\Psi\rangle = \Big(P^aG_{ab}P^b + T^2\det[(G+\mathcal{F})_{ab}]\Big)\,|\Psi\rangle \ . \tag{51}$$

The mass of any physical state of the theory therefore consists of two positive contributions: the first and the second term in eq. (51). For the fermionic constraints to be second class this requirement means that all physical states of the theory have to have strictly non-vanishing mass.

$$M^2|\Psi\rangle \ \geq T^2\det[(G+\mathcal{F})_{ab}]|\Psi\rangle \neq 0|\Psi\rangle \ . \tag{52}$$

This applies equally well to the ground state of the theory. Thus the Hamiltonian analysis of the reason why D-branes admit Lorentz covariant gauge-fixing of κ-symmetry leads to the following conclusion: As long as there are mo massless states in the theory, κ-symmetry admits covariant gauges.

The existence of R-R charges or equivalently the existence of central extensions in supersymmetry algebra (32) is related to $M^2 \neq 0$ condition ($T^2\det[(G+\mathcal{F})_{ab}] \neq 0$) as follows. When $T \neq 0$ and when $\Pi_{am} \neq 0$ and/or $\mathcal{F} \neq 0$, we have simultaneously the R-R charges in the supersymmetry algebra and strictly non-vanishing mass of all physical states in the theory. If $T = 0$ or if $\Pi_{am} = 0$ and $\mathcal{F} = 0$, there are no R-R charges and the massless state is not excluded. However, in this case the covariant gauge is not acceptable, since the fermionic constraints are not invertible.

We may compare this Hamiltonian result with the gauge-fixing of D-branes in static gauges $X^\mu = \sigma^\mu$ which was used in [3]. The tension was equal one there, $T = 1$. In static gauges

$$G_{\mu\nu} = \eta_{\mu\nu} + \dots \ , \qquad \det(G+\mathcal{F})_{ab} = (\det\eta_{ab} + \dots) \neq 0 \ , \tag{53}$$

and therefore both conditions for invertibility of the bracket of covariant fermionic constraints (48) and (49) are satisfied.

7 Covariant quantization of D-0-brane

Consider the κ-symmetric action of a D-0-brane. D-0-brane action does not have Born-Infeld field since there is no place for an antisymmetric tensor of rank 2 in one-dimensional theory. The action (13) for $p = 0$ case reduces to

$$S = -T\left(\int d\tau\sqrt{-G_{\tau\tau}} + \int \bar\theta\Gamma^{11}\dot\theta\right) \ . \tag{54}$$

This action can be derived from the action of the massless 11-dimensional superparticle.

$$S = \int d\tau \sqrt{g_{\tau\tau}} g^{\tau\tau} \left(\dot{X}^{\hat{m}} - \bar{\theta}\Gamma^{\hat{m}}\dot{\theta} \right)^2 \ , \qquad \hat{m} = 0, 1, \cdots, 8, 9, 10. \tag{55}$$

We may solve equation of motion for $X^{\hat{10}}$ as $\mathbf{P}_{\hat{10}} = Z$, where Z is a constant, and use $\Gamma^{11} = \Gamma^{\hat{10}}$. From this one can deduce a first order action

$$S = \int d\tau \left(\mathbf{P}_m (\dot{X}^m - \bar{\theta}\Gamma^m\dot{\theta}) + \frac{1}{2} V(\mathbf{P}^2 + Z^2) - Z\bar{\theta}\Gamma^{11}\dot{\theta} + \bar{\chi}_1 d_2 \right) \ . \tag{56}$$

We will show now that the D-0-brane action can be obtained from this one upon solving equations of motion for $\mathbf{P}_m$, V, χ_1, and d_2. Here $V(\tau)$ is a Lagrange multiplier, $Z = T$ is some constant parameter in front of the WZ term and $\mathbf{P}^2 \equiv \mathbf{P}^m \eta_{mn} \mathbf{P}^n$. The chiral spinors χ_1 and d_2 are auxiliary fields. They are introduced to close the gauge symmetry algebra off shell. To verify that this first order action is one of the D-p-brane family actions given in (13) we can use equations of motion for $\mathbf{P}_m$

$$\mathbf{P}_m = -\frac{1}{V}(\dot{X}^m - \bar{\theta}\Gamma^m\dot{\theta}) \ , \tag{57}$$

and for the auxiliary fields $\chi_1 = 0$ and $d_2 = 0$. The action (56) becomes

$$S = \int d\tau \left(-\frac{1}{2V}(\dot{X}^m - \bar{\theta}\Gamma^m\dot{\theta})^2 + \frac{1}{2} V Z^2 - Z\bar{\theta}\Gamma^{11}\dot{\theta} \right) \ . \tag{58}$$

Equation of motion for V is

$$V^2 = -\frac{1}{Z^2}(\dot{X}^m - \bar{\theta}\Gamma^m\dot{\theta})^2 \ , \tag{59}$$

and we can insert $V = -\frac{1}{Z}\sqrt{-(\dot{X}^m - \bar{\theta}\Gamma^m\dot{\theta})^2}$ back into the action (58) and get

$$S = -Z \left(\int d\tau \sqrt{-(\dot{X}^m - \bar{\theta}\Gamma^m\dot{\theta})^2} + Z\bar{\theta}\Gamma^{11}\dot{\theta} \right) \ . \tag{60}$$

This is the action (13) for D-0-brane at $T = Z$ as given in (54).

The action (56) is invariant under the 16-dimensional irreducible κ-symmetry and under the reparametrization symmetry. The gauge symmetries are (we denote $\Gamma^m \mathbf{P}_m = \not{\mathbf{P}}$):

$$\begin{aligned}
\delta\bar{\theta} &= \bar{\kappa}_2(\Gamma^{11} Z + \not{\mathbf{P}}) \ , && (61)\\
\delta X^m &= -\eta\mathbf{P}^m - \delta\bar{\theta}\Gamma^m\theta - \bar{\kappa}_2\Gamma^m d \ , && (62)\\
\delta V &= \dot{\eta} + 4\bar{\kappa}_2\dot{\theta} + 2\bar{\chi}_1\kappa_2 \ , && (63)\\
\delta\bar{\chi} &= \bar{\kappa}_2\dot{\not{\mathbf{P}}} \ , && (64)\\
\delta d &= [\mathbf{P}^2 + Z^2]\kappa_2 \ . && (65)
\end{aligned}$$

Here $\eta(\tau)$ is the time reparametrization gauge parameter and $\kappa_2(\tau) = \frac{1}{2}(1 - \Gamma^{11})\kappa(\tau)$ is the 16-dimensional parameter of κ-symmetry. The gauge symmetries form a closed algebra

$$[\delta(\kappa_2), \delta(\kappa_2')] = \delta(\eta = 2\bar{\kappa}_2\not{\mathbf{P}}\kappa_2') \ . \tag{66}$$

To bring the theory to the canonical form we introduce canonical momenta to θ and to V and find, excluding auxiliary fields

$$L = \mathbf{P}_m \dot{X}^m + P_V \dot{V} + \bar{P}_\theta \dot{\theta} + \frac{1}{2} V(\mathbf{P}^2 + Z^2) + P_V \varphi + \left(\bar{P}_\theta + \bar{\theta}(\not{\mathbf{P}} + Z\Gamma^{11})\right) \psi \ . \tag{67}$$

We have primary constraints $\bar{\Phi} \equiv \bar{P}_\theta + \bar{\theta}(\not{\mathbf{P}} + Z\Gamma^{11}) \approx 0$ and $P_V = 0$. The Poisson brackets for 32 fermionic constraints are

$$\{\Phi, \Phi\} = 2C(\not{\mathbf{P}} + \Gamma_{11} Z) \ . \tag{68}$$

We also have to require that the constraints are consistent with the time evolution $\{P_V, H\} = 0$. This generates a secondary constraint

$$t = \mathbf{P}^2 + Z^2 \ . \tag{69}$$

Thus the Hamiltonian is weakly zero and any physical state of the system satisfying the reparametrization constraint is a BPS state $M = |Z|$ since

$$\mathbf{P}^2 + Z^2 |\Psi\rangle = 0 \qquad \Longrightarrow \qquad Z^2|\Psi\rangle = -\mathbf{P}^2|\Psi\rangle = M^2|\Psi\rangle \ . \tag{70}$$

The 32×32 -dimensional matrix $C(\not{\mathbf{P}} + \Gamma_{11} Z)$ is not invertible since it squares to zero when the reparametrization constraint is imposed. This is a reminder of the fact that D-0-brane is a d=11 massless superparticle. The 32 dimensional fermionic constraint has a 16-dimensional part which forms a first class constraint and another 16-dimensional part which forms a second class constraint. We notice that the Poisson brackets reproduce the $d = 10$, $N = 2$ algebra with the central charge which can also be understood as $d = 11$, $N = 1$ supersymmetry algebra with the constant value of $\mathbf{P}_{11} = Z$.

We proceed with the quantization and gauge-fix κ-symmetry covariantly by taking $\theta_2 = 0, \theta_1 \equiv \lambda$ and find

$$L^\kappa_{g.f.} = \mathbf{P}_m(\dot{X}^m - \bar{\lambda}\Gamma^m \dot{\lambda}) + \frac{1}{2} V(\mathbf{P}^2 + Z^2) \ . \tag{71}$$

The 16-dimensional fermionic constraint

$$\bar{\Phi}_\lambda \equiv (\bar{P}_\lambda + \bar{\lambda}\not{\mathbf{P}}) \approx 0 \tag{72}$$

forms the Poisson bracket

$$\{\Phi^\alpha_\lambda, \Phi^\beta_\lambda\} = 2(\not{\mathbf{P}} C)^{\alpha\beta} \ . \tag{73}$$

The matrix $\not{\mathbf{P}} C$ is perfectly invertible as long as the central charge Z is not vanishing. The inverse to (73) is

$$\{\Phi^\alpha, \Phi^\beta\}^{-1} |_{t=0} = [2(\not{\mathbf{P}} C)^{\alpha\beta}]^{-1} = \frac{(C\not{\mathbf{P}})_{\alpha\beta}}{2\mathbf{P}^2} \ . \tag{74}$$

This proves that the fermionic constraints are second class and that the fermionic part of the Lagrangian

$$-\bar{\lambda}\not{\mathbf{P}}\dot{\lambda} \equiv -i\lambda^\alpha \Phi_{\alpha\beta} \dot{\lambda}^\beta \ , \qquad \Phi_{\alpha\beta} = -i(C\not{\mathbf{P}})_{\alpha\beta} \ , \tag{75}$$

is not degenerate in a Lorentz covariant gauge. None of this would be true for a vanishing central charge. Note that in the rest frame $\mathbf{P}_0 = M, \vec{\mathbf{P}} = 0$, hence

$$\Phi_{\alpha\beta} = M\delta_{\alpha\beta} \ . \tag{76}$$

For D-0-brane one can covariantly gauge-fix the reparametrization symmetry by choosing the $V = 1$ gauge and including the anticommuting reparametrization ghosts b, c. This brings us to the following form of the action:

$$L^{\kappa,\eta}_{g.f.} = \mathbf{P}_m \dot{X}^m - \bar{\lambda}\not{\mathbf{P}}\dot{\lambda} + \frac{1}{2}(\mathbf{P}^2 + Z^2) + b\dot{c} \ . \tag{77}$$

Now we can define Dirac brackets

$$\{\lambda, \bar{\lambda}\}^* = \{\lambda, \bar{\Phi}\}\{\bar{\Phi}, \Phi\}^{-1}\{\Phi \ , \bar{\lambda}\} = \frac{\not{\mathbf{P}}}{2\mathbf{P}^2} = -\frac{\not{\mathbf{P}}}{2Z^2} \ . \tag{78}$$

The generator of the 32-dimensional supersymmetry is

$$\bar{\epsilon}Q = \bar{\epsilon}(\not{\mathbf{P}} + \Gamma^{11}Z)\lambda \ . \tag{79}$$

It forms the following Dirac bracket

$$[\bar{\epsilon}Q \ , \bar{Q}\epsilon']^* = \bar{\epsilon}(\not{\mathbf{P}} + \Gamma^{11}Z)\frac{\not{\mathbf{P}}}{2\mathbf{P}^2}(\not{\mathbf{P}} + \Gamma^{11}Z)\epsilon' = \bar{\epsilon}\Gamma^{\hat{m}}\mathbf{P}_{\hat{m}}\epsilon' = \bar{\epsilon}(\not{\mathbf{P}} + \Gamma^{11}Z)\epsilon' \ . \tag{80}$$

We can also rewrite it in d=11 Lorentz covariant form

$$[\bar{\epsilon}Q \ , \bar{Q}\epsilon']^* = \bar{\epsilon}\Gamma^{\hat{m}}\mathbf{P}_{\hat{m}}\epsilon' = \bar{\epsilon}\hat{\not{\mathbf{P}}}\epsilon' \ , \qquad \hat{m} = 0, 1, \cdots, 8, 9, 10, \qquad Z = \mathbf{P}_{\hat{10}} \ , \qquad \Gamma^{11} = \Gamma^{\hat{10}} \ . \tag{81}$$

This Dirac bracket realizes the d=11, N=1 supersymmetry algebra or, equivalently, d=10, N=2 supersymmetry algebra with the central charge Z.

One can also to take into account that the path integral in presence of second class constraints has an additional term with $\sqrt{\mathrm{Ber}\{\Phi_\lambda, \Phi_\lambda\}} \sim \sqrt{\mathrm{Ber}\,\Phi_{\alpha\beta}}$ [12], see Appendix. It can be used to make a change of variables

$$S_\alpha = \Phi^{1/2}_{\alpha\beta} \ \lambda^\beta \ . \tag{82}$$

The action becomes

$$L = \mathbf{P}_m \dot{X}^m - iS_\alpha \dot{S}_\alpha + b\dot{c} - H \tag{83}$$

$$H = -\frac{1}{2}(\mathbf{P}^2 + Z^2) \ . \tag{84}$$

The generators of global supersymmetry commuting with the Hamiltonian take the form

$$\bar{\epsilon}Q = \bar{\epsilon}(\not{\mathbf{P}} + \Gamma^{11}Z)\Phi^{-1/2}S \ . \tag{85}$$

Taking into account that $\{S_\alpha, S_\beta\}^* = -\frac{i}{2}\delta_{\alpha\beta}$ we have again realized $d = 10$, $N = 2$ supersymmetry algebra in the form (80) or (81).

The terms with anticommuting fields S_α can be rewritten in a form where it is clear that they can be interpreted as world-line spinors,

$$L = \mathbf{P}_m \partial_0 X^m + \bar{S}_\alpha \rho^0 \partial_0 S_\alpha + b\dot{c} - H \ . \tag{86}$$

Here $\bar{S}_\alpha = iS_\alpha \rho^0$ and $(\rho^0)^2 = -1$, $\rho^0 = i$ being a 1-dimensional matrix.

Thus, we have the original 10 coordinates X^m and their conjugate momenta $\mathbf{P}_m$, and a pair of reparametrization ghosts. There are also 16 anticommuting world-line spinors S, describing 8 fermionic degrees of freedom. The Hamiltonian is quadratic. The ground state with $M^2 = Z^2$ is the state with the minimal value of the Hamiltonian. Thus for the D-superparticle one can see that the condition for the covariant quantization is satisfied in the presence of a central charge which makes the mass of a physical state non-vanishing. The global supersymmetry algebra is realized in a covariant way, as different from the light-cone gauge.

8 Conclusion

Thus, we have confirmed here the conclusion of the work [3] that one can covariantly quantize D-p-branes. However, as different from [3], we did not use the static gauge for fixing the reparametrization symmetry, and analysed the Hamiltonian structure of the theory. This allowed us to clarify the reason and the generic condition under which a covariant quantization of D-p-branes is possible: the ground state has to be strictly massive, $M^2_{\text{groundstate}} = Z^2 > 0$. Technically, this reduces to 2 conditions:

i) The tension of the D-p-brane T has to be non-vanishing.

ii) $\det(G + \mathcal{F})_{ab}$ has to be non-vanishing (a, b are space components of the brane).

Those two conditions for the D-p-brane are basically equivalent to the definition of this object. Both these conditions have to be satisfied if there are non-vanishing central charges in the supersymmetry algebra which are due to the existence of R-R charges of the D-p-brane. The cross term in the left-right part of N=2 supersymmetry algebra

$$\{Q_\alpha,\ \tilde{Q}_\beta\} = 2\sum_A (C\Gamma^A)_{\alpha\beta} T \frac{Q^R_A}{p!} \tag{87}$$

does not vanish when conditions i) and ii) for covariant quantization are satisfied. These conditions provide the *invertibility of the second class fermionic constraints in Lorentz covariant gauges*.

As the special case we have performed a covariant quantization of D-0-brane. The resulting supersymmetry generator is $d = 10$ Lorentz covariant and the Dirac bracket of the quantized theory form $d = 10$, $N = 2$ supersymmetry algebra with a central charge.

By announcing that D1-brane can be covariantly quantized we have to be able to deal covariantly with the IIB Green-Schwarz action since it is SL(2,R) dual to the D1-brane. Our conclusion is that this is indeed possible under the condition that the massless state is projected out from the fundamental IIB string. In [3] this was effectively demonstrated since the static gauge used there corresponds to the IIB superstring wrapped around the circle and such object does not have massless excitations. Thus, we conclude that the old problem of impossibility to covariantly quantize the fundamental string can be avoided for IIB fundamental string for all states but massless. The dual partner of it, D-1 string, does not have massless states, and duality works only in the sector of massive excitations of these two theories, quantized covariantly. To confirm this picture and better understand these issues one has to quantize these two dual theories in the conformal gauge for the reparametrization symmetry and in covariant gauge for κ-symmetry.

One has to note here that the choice of the Lorentz covariant fermionic gauge is not trivial for objects dual to D-branes. It has been explained in [10] that the projector Γ of κ-symmetry of D-p-branes anticommutes with Lorentz covariant chiral projectors in d=10. This technically explains why Lorentz covariant gauges are capable of removing the degeneracy of the theory due to κ-symmetry on D-p-branes. This is not the case for the fundamental strings, and the choice of the acceptable Lorentz covariant fermionic gauge has to be done properly. It is important that in IIB theory with two chiral spinors any gauge of the type

$$\theta_1 = c\theta_2 \ , \tag{88}$$

where c is and arbitrary constant, is consistent with Lorentz symmetry.

Similar observations apply to the D2-brane versus the eleven-dimensional supermembrane [6] which are also related to each other, this time via duality on the worldvolume. Is it possible that we can covariantly quantize the D2-brane but not the dual d=11 supermembrane? First notice that the chiral projectors $\frac{1}{2}(1 \pm \Gamma^{11})$ which have been used for the covariant quantization of a D-2-brane are covariant in d=10 but not Lorentz covariant in d=11. Secondly, our requirement of the absence of massless states in d=10 for the D-2-brane does not exclude the massless d=11 state since we have a BPS ground state in d=10. As we have seen in quantization of the D-0-brane in Sec. 7, the BPS condition $-\mathbf{P}_{10}^2 = M_{10}^2 = Z^2$ is in fact equivalent to the statement that the 11-dimensional superparticle is massless, $-\mathbf{P}_{10}^2 - Z^2 = -\mathbf{P}_{11}^2 = 0$.

At this stage we have learned that the 10d Lorentz covariant quantization of the D-2-brane may be a step towards understanding of the spectrum of states of the fundamental theory. In particular, we have found the mass formula for the D-2-brane (8) which is a d=10 Lorentz covariant generalization of the mass formula of the supermembrane quantized in the light-cone gauge [7, 8].

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Appendix: Quantization in canonical gauges

Quantization of an arbitrary Bose-Fermi-system with the first and second class constraints in canonical gauges was performed by E. Fradkin and collaborators [12]. One starts with the classical Lagrangian of the form

$$L = p_i \dot{q}^i - H_0(p,q) - \xi^k \Theta_k(p,q) - \xi^\mu T_\mu(p,q) \ . \tag{89}$$

The first class constraints obey the relations

$$\{T_\mu, T_\nu\} \mid_{T=0,\Theta=0} = 0 \ , \qquad \{H_0 \ , T_\mu\} \mid_{T=0,\Theta=0} = 0 \ , \tag{90}$$

and for the second class constraints we have

$$\mathrm{Ber}\{\Theta_k, \Theta_l\} \mid_{T=0,\Theta=0} \neq 0 \ . \tag{91}$$

The symbol $\{\}$ stands for the Fermi-Bose extension of the Poisson bracket and Ber (Berezinian) is a superdeterminant of the matrix $\{\Theta_k, \Theta_l\} \equiv \Theta_{kl}$. The Dirac bracket is defined as

$$.\{A, B\}^* = \{A, B\} - \{A, \Theta_k\}(\Theta_{kl})^{-1}\{\Theta_l, B\} \ . \tag{92}$$

The path integral in canonical gauges $\Phi^\mu(p,q) = 0$ (where all ghosts are non-propagating fields) is given by

$$Z = \int \exp\{iS[q,p,\xi,\pi]\} \prod (\mathrm{Ber}\{\Phi, T\}^* \delta(\Theta)(\mathrm{Ber}\{\Theta,\Theta\})^{1/2} dq \ dp \ d\xi d\pi) \ , \tag{93}$$

where the action is

$$S = \int (p_i \dot{q}^i - H_0(p,q) - \xi^\mu T_\mu(p,q) - \pi_\mu \Phi^\mu) d\tau \ . \tag{94}$$

References

[1] J. Polchinski, *TASI Lectures on D-branes*, hep-th/9611050.

[2] M. Cederwall, A. von Gussich, B.E.W. Nilsson, and A. Westerberg, *The Dirichlet Super Three Brane In Ten-Dimensional Type IIIB Supergravity*, Nucl. Phys. **B490**, 163 (1997);
M. Cederwall, A. von Gussich, B.E.W. Nilsson, P. Sundell, and A. Westerberg, *The Dirichlet Super P-Branes In Ten-Dimensional Type IIA And IIB Supergravity*, Nucl. Phys. **B490**, 179 (1997);
M. Cederwall, *Aspects of D-brane actions*, Report GOTEBORG-ITP-96-20, hep-th/9612153.

[3] M. Aganagic, C. Popescu, and J.H. Schwarz, *D-Brane Actions with Local Kappa Symmetry*, Phys. Lett. **B393**, 311 (1997), hep-th/9610249;
M. Aganagic, C. Popescu, and J.H. Schwarz, *Gauge Invariant and Gauge Fixed D-brane Actions*, Report CALT-68-2088, hep-th/9612080;
M. Aganagic, J. Park, C. Popescu, and J.H. Schwarz, *Dual D-brane Actions*, Report CALT-68-2099, hep-th/9702133.

[4] E. Bergshoeff and P.K. Townsend, *Super D-Branes*, Nucl. Phys. **B490**, 145 (1997), hep-th/9611173.

[5] M. Abou Zeid and C. M. Hull, *Intrinsic geometry of D-branes*, hep-th/9704021.

[6] E. Bergshoeff, E. Sezgin, and P.K. Townsend, Phys. Lett. **189B**, 75 (1987).

[7] E.A. Bergshoeff, E. Sezgin, and Y. Tanii, Nucl. Phys. **B298**, 187 (1988).

[8] B. de Wit, J. Hoppe, and H. Nicolai, Nucl. Phys. **B305** [FS23], 545 (1988); B. de Wit, M. Lüscher and H. Nicolai, Nucl. Phys. **B320**, 135 (1989).

[9] T. Banks, W. Fischler, S.H. Shenker, and L. Susskind, *M-theory as a matrix model: a conjecture*, hep-th/9610043.

[10] E. Bergshoeff, R. Kallosh, T. Ortín, and G. Papadopoulos, *κ-symmetry, Supersymmetry and Intersecting branes*, CERN–TH/97–87, hep-th/9705040.

[11] U. Lindström and R. von Unge, *A Picture of D-branes at Strong Coupling*, hep-th/9704051.

[12] E. S. Fradkin, Acta Universitatis Wratislaviensis **207**. Proceedings of X-th Winter School in Karpach, 1973;
E. S. Fradkin and G.A. Vilkovisky, Phys. Lett. **55B**, 224 (1975);
I.A. Batalin and G.A. Vilkovisky, Phys. Lett. **69B**, 309 (1977);
E. S. Fradkin and T.E. Fradkina, Phys. Lett. **72B**, 343 (1978).